# Optimal Green Certificate Auction Design for the Electricity Sector

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## Abstract

With the highly development of the advanced carbon capture technology, the industrialization and widely application could be in the coming future. The cost of the technology operation could directly be paid by the revenue of a well-designed carbon market. In this paper, we proposed a green certificate auction which aims to earn more for the carbon capture. We consider that each generator have willingness to contribute to the carbon capture and it will balance it with his actual benefit in economic dispatch. We show that our mechanism satisfies optimality, truthfulness and individual rationality. We also show our framework could work enough well for many rounds even if the willingness could not be well identified. We also conduct numerical studies to show the effectiveness of our proposed theoretical approximation. Our work could be implemented in the real market to guide the firms to better make contribution to the carbon capturing, which could also be the resources of the green certificate. We could also design the mechanism to assign reward for the contribution in the future work.

## 1 Introduction

Global warming is coming [1]. The Paris Agreement [2] calls for more active efforts from all over the world to combat global warming by reducing the carbon emissions. To achieve this goal, carbon tax and cap-and-trade programs are ready in many countries for large scale implementation. However, such policies often make the regions with such policies in a non-favorable position in the global economy. More importantly, the public often challenge the actual usage of the extra payment collected from such carbon related policies. We note that carbon capture technologies are becoming mature, and some of them have been successfully commercialized [3]. Hence, we imagine that one potential solution to align the interests between the public and the policy maker is to invest the extra payment in carbon capture technologies or to advance the large scale deployment of the commercialized technologies. In this work, we seek to design the green certificate auction to maximize the revenue for the auctioneer (i.e., the system operator in the power grid).

Specifically, we consider the green certificate auction design together with economic dispatch (ED), a classic procedure in the electricity sector to dispatch the generators to meet the real time demand. The designed auction will determine the social allocations of green certificates, which will grant the generating companies the dispatch opportunities in the ED process. We seek to design the most effective auction to better enable the environmental protection.

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## 1.1 Related Works

We identify two closely related research streams. The first one investigates the financial instruments applications to the carbon market. The other one is the theoretical treatment for homogeneous divisible goods' auction.

Various financial instruments have been implemented in the carbon related markets, e.g., auction, grandfathering, uniformly or discriminatory pricing [4]. We focus on the auction design, which is also the most popular form compared with its rivals [5]. In the literature, both sealed auctions and dynamic auctions have been designed for carbon allowance allocation [6]. For example, in [7], Betz *et al.* propose an ascending clock auction to improve the efficiency of the carbon permit market; Wang *et al.* employ the sequential ascending auction and proving its convergence to the Pareto optimal equilibrium in [8]. In [9], Rao *et al.* study the uniform price sealed auction and show there exists an asymmetric Nash Equilibrium. Sun *et al.* generalize the setting by considering the multi-buyers and multi-sellers scenario, and design a double action for carbon permit allocation in [4]. Ding *et al.* take into account the influence of the interactions in the permit auction and design a two-stage auction-bargaining model in [10]. Wang *et al.* design the multi-unit auction in the Bayesian framework in [11]. Overall, the literature seldom treat the problem as a divisible good auction and seldom involve rigorous modeling of the whole dispatch process. Furthermore, the literature often designs the market for the policy maker, instead of the system operator in the electricity sector. In contrast, these are the focuses in our work.

We cast the carbon allowance auction in the homogeneous divisible goods auction design framework, since the carbon permissions (i.e., green certificates) are homogeneous in nature. Such an auction is often organized in two ways. The first approach is to discretize the quantity space to a countable number [12]. However, when the number of pieces resulting from the division is too large, the effective auction design is an open problem. Another approach is pioneered by Wilson in [13], which designs an effective bidding in a stylized model. This classical work has been applied to both the divisible goods auction [14, 15], without specifying the difference between uniform auction and discriminatory auction. Back *et al.* in [16] further show that discriminatory auction could yield more revuenue in divisible goods auction. Recently, Lu *et al.* design the divisible unit good auction with budget constraints in [17]. Johari *et al.* propose a scalar strategy Vickrey-Clarke-Groves (SSVCG) mechanism and introduce an efficient algorithm to characterize the Nash Equilibrium in [18]. However, most of these works assume the knowledge on the bidders' value's distribution. Sample complexity [19] is proposed to handle such kind of issues. Specifically, Dhangwatnotai *et al.* and [20] and Cole *et al.* [21] both study the sample complexity for digital goods with the unlimited supply. In this work, we first follow the classical framework proposed in [14] and then use the notion of sample complexity to infer the generating companies' valuation with limited supply of green certificates.

## 1.2 Our Contributions

In seek of designing the effective green certificate auction in the Bayesian framework, our principle contributions can be described as follows:

1. *Virtual Demand*: We study the coupling between green certificate auction and the ED process. By introducing a decoupling algorithm, we employ the notion of virtual demand, inspired by the virtual value in the classical Myerson's auction [22].
2. *Performance Evaluation*: In the Bayesian framework, we submit our designed auction is both truthful and individually rational. Furthermore, we investigate the impact of incorporating load constraint in our auction design, in both competitive and non-competitive scenarios.
3. *Sample Complexity*: We conduct sample complexity analysis for our auction design and construct an upper bound for the number of samples needed to estimate the value of bidders. This could help us better understand when the auction design can achieve good performance.

The rest of the paper is organized as follows. In Section 2, we formulate the green certificate auction design problem. Then, we propose our designed auction in the constraint free setting and prove its effectiveness in Section 3. Next, we extend our designed auction with a load constraint in Section 4. To relieve the assumption on the knowledge of bidder value's distribution, we conduct a sample complexity analysis in Section 5. After that, numerical studies are conducted to verify our conclusions in Section 6. Finally, concluding remarks are given in Section 7.

## 2 Auction Formulation

We consider the optimal multi-unit auction for green certificate. We assume the green certificates are divisible, with a total amount of $Q$. The bidders in the auction are the generating companies whereas the auctioneer is the system operator. Denote the total number of bidders by $N$, and the type of generating company $i$ by $v_i$. This type information characterizes the company's willingness by holding the certificate. It could come from future trading, reputation or other side reward beyond this mechanism. Specifically, denote the auction outcome for each generating company $i$ by variable $x_i$. We focus on studying the uniform demand price function, i.e., the price $r(x_i, v_i)$ is fully characterized by the auction outcome $x_i$ and type $v_i$. Note that the integral of $r(x_i, v_i)$ with respect to $x_i$ describes the valuation for generating company $i$. Mathematically, if the realized auction outcome for generating company $i$ (green certificate purchased by the generating company through the auction) is $q_i$, then its valuation $N(q_i, v_i)$ can be calculated as follows:

$$N(q_i, v_i) = \int_0^{q_i} r(x, v_i)dx. \tag{1}$$

We further assume the type information $v_i$ is drawn from the cumulative probability function (c.d.f.) $F_i(v)$ with a support of $[\underline{v_i}, \overline{v_i}]$. Denote its corresponding probability density function (p.d.f.) by $f_i$.

To simplify the optimal auction design, we make the following technical assumptions:

- **A1**: all the type distributions of bidders are regular. That is, for each generating company $i$,

  $$J_i(v) = v - \frac{1}{\rho_i(v)} \tag{2}$$

  is increasing, where $\rho_i(v)$ represents the hazard rate for the bidder $i$, which is defined as follows:

  $$\rho_i(v) = f_i(v)/[1 - F_i(v)]. \tag{3}$$

- **A2**: For each type $v$, $r(x, v)$ is finite, twice continuously differentiable, strictly decreasing in $x$, and strictly increasing in $v$ when $r$ is greater than zero.
- **A3**: The elasticity of the demand price function is non-decreasing, i.e.,

  $$\frac{\partial}{\partial v}(-\frac{x}{r}\frac{\partial r}{\partial x}) \leq 0. \tag{4}$$

- **A4**: The demand price function is concave in $v$, i.e.,

  $$\frac{\partial^2 r}{\partial v^2} \leq 0. \tag{5}$$

Assumption A1 is often used in economics to guarantee the virtual valuation's monotonicity and incentive compatibility in Myerson's auction [22]. The next three assumptions are standard technical assumptions for demand price functions. And large classes of preferences satisfying above four assumptions [23].

With these four assumptions, we can formulate the ED process, which is essential in characterizing the objective functions for both the bidders and the auctioneer in the green certificate auction.

Assume the marginal cost of generating company $i$ to be $\alpha_i$. Denote the total energy demand by $d$ and the energy generation of generator $i$ by $g_i$. Clearly, this generation level is bounded by the generation capacity constraint. Specifically, in an emission aware ED, we assume this generation level, without the purchase of green certificate, is bounded by $B_i$. Green certificate is used to grant the generators more opportunities to be dispatched in ED. For example, if generating company $i$, though the auction, obtains $q_i$ amount of green certificates, its maximal generation level becomes $B_i + q_i$. To ensure there exists a feasible solution, we require $d \leq \sum_{i=1}^{N} B_i$. Thus, the system operator, based on the outcome of the auction, could conduct the ED by solving the following optimization problem:

$$
\begin{aligned}
\text{(P1)}\ \min & \sum_{i=1}^{N} \alpha_i g_i \\
s.t.\ & \sum_{i=1}^{N} g_i = d \\
& 0 \leq g_i \leq B_i + q_i \quad \forall i.
\end{aligned}
\tag{6}
$$

The first constraint is the supply demand balance constraint, and the second set of constraints refer to the generation capacity constraints. This optimization problem decides the energy price $\lambda$, which is the Lagrangian multiplier associated with the supply demand balance constraint. Define $\lambda_0$ to be the energy price without the green certificate auction (i.e., all the $q_i$'s are zero). We can study the extra profit for generating company $i$ by participating the auction:

$$
\begin{aligned}
V_i &= \mathbf{I}(\alpha_i < \lambda)\lambda(g_i - B_i) + \mathbf{I}(\alpha_i < \lambda)(\lambda - \lambda_0)B_i \\
&\quad + \mathbf{I}(\lambda \leq \alpha_i < \lambda_0)\lambda_0 B_i \\
&= \mathbf{I}(\alpha_i < \lambda)\lambda(g_i - B_i) + \phi(\lambda, \lambda_0, \alpha_i)
\end{aligned}
\tag{7}
$$

where $\mathbf{I}(\cdot)$ is the indicator function.

Characterizing the ED process sharps our understanding of the auction design, as these two processes are closely coupled together. From the ED, the objective of the system operator is to minimize the extra payment for the certificate in ED as well as to maximize the auction revenue. We represent this objective function as follows:

$$
\max \quad \mathbf{E}_{v_i}\left[\sum_{i=1}^{N} p_i - \sum_{i=1}^{N} V_i\right], \tag{8}
$$

where $p_i$ denotes the payment that the bidder $i$ pays for $q_i$ green certificates.

Each generating company's utility function also consists of two parts: one is the utility extracted from the auction and another is the profit from ED[1]. We denote the utility function by $U_i$:

$$
U_i = N(q_i, v_i) - p_i + V_i. \tag{9}
$$

We assume that if participating in the auction does not provide the generating company any extra profit, it would not participate in the auction.

## 3 Revenue-Maximizing Pricing Scheme Design

This problem could be considered as a variant of Myerson auction. Our problem needs to consider ED problem which could be influenced by the results of auction. Therefore, to decouple this problem, we need to find a more direct formulation of ED problem first. Notice that in our problem the final solution for $\lambda$ could come from a discrete set $\{\alpha_i, i \in [N]\}$.

Then we fix $\lambda$ as $\alpha_i$ then conduct the auction. We could directly know the best $g_i$ for generator $i$ as follows:

$$
g_i = \begin{cases} 0 & \lambda \leq \alpha_i \\ B_i + q_i & \lambda > \alpha_i \end{cases} \tag{10}
$$

Then we could further know with the actual utility function $U_i^\lambda$ with ED price $\lambda$ for generator $i$ as follows:

$$
U_i^\lambda = \begin{cases} N(q_i, v_i) - p_i + \phi(\lambda, \lambda_0, \alpha_i) & \lambda \leq \alpha_i \\ N(q_i, v_i) - p_i + (\lambda - \alpha_i)q_i + \phi(\lambda, \lambda_0, \alpha_i) & \lambda > \alpha_i \end{cases} \tag{11}
$$

[1] For simplification, we assume that for the generating company $i$, whose marginal cost equals to the price, it will not take part in ED.

Then we could also construct the virtual demand function $I^{\lambda}$ as follows:

$$I^{\lambda}(q_i, v_i, \alpha_i) = \begin{cases} N(q_i, v_i) - \dfrac{1}{\rho_i(v_i)}\dfrac{\partial N(q_i, v_i)}{\partial v_i} & \lambda \leq \alpha_i \\ N(q_i, v_i) - \dfrac{1}{\rho_i(v_i)}\dfrac{\partial N(q_i, v_i)}{\partial v_i} - \alpha_i q_i & \lambda > \alpha_i \end{cases} \tag{12}$$

Here, we back to our rare items assumption. More, specifically we assume that for all $v_i$ profiles, we define $q_i^{'}$ as the amount to make $\frac{\partial I^{\lambda}(q_i, v_i, \alpha_i)}{\partial q_i} = 0$ when $\lambda > \alpha_i$. Our scarcity assumption shows that $\sum_{i=1}^{N} q_i^{'} \geq Q$, otherwise there could exist $i$ that makes $\frac{\partial I^{\lambda}(q_i, v_i, \alpha_i)}{\partial q_i} > 0$, which shows all the bidders need to compete for the qualification.

Then we need to design an optimal multi-unit auction for agents. In our framework, the bidder would bid its own $\theta_i$ to the system operator and the system operator would provide a recipe with $(p_i, q_i)$ for the allocation and payment. We construct the optimization problem to derive the best allocation and payment. The problem could be made as follows:

$$\begin{aligned} \text{(P2)} \ \max_{q_i} \sum_{i=1}^{N} I^{\lambda}(q_i, v_i, \alpha_i) \\ s.t. \quad \sum_{i=1}^{N} q_i \leq Q \end{aligned} \tag{13}$$

Then we could construct the conditions for the optimal solutions of the problem through K.K.T. conditions as follows:

$$\begin{aligned} q_i^{*}[\frac{\partial I}{\partial q_i} - \mu] = 0 \\ \sum_{i=1}^{N} q_i \leq Q \\ q_i^{*} = 0 \rightarrow \frac{\partial I}{\partial q_i}(0, v_i) \leq \mu, \end{aligned} \tag{14}$$

where $q_i^{*}$ denote the optimal allocation for bidder $i$.

Then after finding the answers for Eqs. 14, we need to construct the allocation $p_i$ as follows:

$$p_i = N(q_i, v_i) - \frac{1}{\rho_i(v_i)}\frac{\partial N(q_i, v_i)}{\partial v_i} + \mathbf{I}(\alpha_i < \lambda)(\lambda - \alpha_i)q_i + \phi(\lambda, \lambda_0, \alpha_i) \tag{15}$$

With the construction above, our auction algorithm bases on the optimal multi-unit auction is showed in Algorithm 1.

In our algorithm, we actually conduct an optimal multi-unit auction for the certificate. We could first assume $\lambda$ and try to find the possible realized $\lambda$ for the following ED process. Since the $\lambda$ is from a discrete set in our problem, we could simply traverse all possible $\lambda$.

Actually, we need to check the correctness and the effectiveness of our proposed auction. We conclude it in the following two theorems.

**Theorem 1** There must exist a $\lambda$ that satisfies the demand $d$.

**Proof:** In this part, we want to show that we could find out $\lambda$ that satisfies the demand $d$.

First, we sort the generator according to the cost $\alpha_i$. Without loss of generality, we denote it as $\alpha_1, ..., \alpha_N$ and we rank the generator with this order. Then if for $\lambda = \alpha_K$, we could not satisfy the demand, that is $d > \sum_{i=1}^{K-1} B_i + \sum_{i=1}^{K-1} q_i$, here we use $q_i$ to denote the allocation for each generator. Then it's obvious that $\sum_{i=1}^{K-1} q_i \leq Q$, if $\lambda$ increase, we could denote it as $\lambda' = \alpha_{K+1}$. We could find out that the $I^{\lambda}$ for this $K-1$ bidders could not change. For $K$-th bidder, $I^{\lambda}$ needs to

**Algorithm 1:** Green Certificate Auction Considering ED Process

**Input:** The generator $i$'s cost $\alpha_i$ $\forall i \in [N]$;
The generator $i$'s bidding type $v_i$ $\forall i \in [N]$;
**Output:** The final allocation $q_i$ $\forall i \in [N]$;
The price $\lambda$ for ED process;
The payment $p_i$ $\forall i \in [N]$;

Initialize $R = 0$
**for** $\lambda \in \{\alpha_1, ..., \alpha_N\}$ **do**
  Solve the optimization problem Eqs. 13 and 15 with the conditions Eq. 14 to derive $(q_i^{\lambda}, p_i^{\lambda})$ $\forall i$.
  Use final auction allocation results and price $\lambda$ to conduct ED process
  **if** $d$ is exactly satisfied **then**
    Calculate the final revenue $R_t$ for auctioneer;
    $q_i = q_i^{\lambda}$,$p_i = p_i^{\lambda}$ $\forall i$; $R = R_t$
  **end if**
**end for**
**return** $(q_i, p_i)$ $\forall i$; $\lambda$;

take out $\alpha_K q_K$. From Eqs. 14, we could derive that the new auction price $\mu^{'}$ should not be higher than the original $\mu$. Therefore if we denote the new allocation for bidder $i$ as $q_i^*$, we could find that $\sum_{i=1}^{K-1} q_i^* \geq \sum_{i=1}^{K-1} q_i$. Therefore, $\sum_{i=1}^{K} B_i + \sum_{i=1}^{K} q_i$ could increase with $K$. Then we could also derive from our assumption that $d \leq \sum_{i=1}^{N} B_i + Q$, which shows that our largest possible generation could be higher than demand. Then since $\sum_{i=1}^{K} B_i + Q$ is strictly increasing by $\lambda$, we could find the $\lambda$ that exactly satisfies demand $d$.

Then we turn to show the effectiveness of our mechanism. We define the bidder $i$'s strategy as $s_i(v_i)$ for bidding. Then we define the $s = [s_1, ..., s_N]$ and $s_{-i} = [s_1, ...s_{i-1}, s_{i+1}, ..., s_N]$ and we could also define the Bayesian equilibrium profile as $s^* = [s_1^*, ..., s_N^*]$ where $s_i^*(\cdot)$ is equilibrium strategy. We assume our allocation is deterministic and we could derive $q_i(x, v_{-i}) = q_i(s_i^*(x), s_{-i}^*(v_{-i}))$ and $p_i(x) = \mathbf{E}_{v_{-i}} p_i(s_i^*(x), s_{-i}^*(v_{-i}))$, where $x$ represents the type that bidder with type $v_i$ plays on. We also denote that $q_{-i}(x, v_{-i}) = [q_1(s_i^*(x), s_{-i}^*(v_{-i})), ..., q_{i-1}(s_i^*(x), s_{-i}^*(v_{-i})), q_{i+1}(s_i^*(x), s_{-i}^*(v_{-i})), ..., q_N(s_i^*(x), s_{-i}^*(v_{-i}))]$.

Then we also define the expected surplus for the generator $i$ $\Pi_i(x, v_i)$ as follows:

$$
\begin{aligned}
\Pi_i(x, v_i) =& \mathbf{E}_{v_{-i}} N(q_i(x, v_{-i}), v_i) - p_i(x) \\
&+ \mathbf{E}_{v_{-i}} (\lambda(q_i(x, v_{-i}), q_{-i}(x, v_{-i})) - \alpha_i)^+ q_i(x, v_{-i}) \\
&+ \mathbf{E}_{v_{-i}} \phi(\lambda(q_i(x, v_{-i}), q_{-i}(x, v_{-i})), \lambda_0, \alpha_i) \\
=& \mathbf{E}_{v_{-i}} N(q_i(x, v_{-i}), v_i) - p_i(x) \\
&+ \mathbf{E}_{v_{-i}} (\lambda(x, v_{-i}) - \alpha_i)^+ q_i(x, v_{-i}) \\
&+ \mathbf{E}_{v_{-i}} \phi(\lambda(x, v_{-i}), \lambda_0, \alpha_i),
\end{aligned}
\tag{16}
$$

where $(\cdot)^+$ denote $\max(\cdot, 0)$.

Then we define both Bayesian Incentive Compatibility (BIC) and Interim Individual Rationality (IR).

**Definition 1**(BIC) Since $s_i^*(v_i)$ could be the best strategy, the Bayesian Incentive Compatibility guarantees that

$$\Pi_i(v_i, v_i) = \max_x \Pi_i(x, v_i)$$

**Definition 2**(Interim IR) We define Interim Individual Rationality as $\Pi_i(v_i, v_i) \geq 0$ since we assume the generators are Rational individual.

**Theorem 2** Our auction could satisfy Bayesian Incentive Compatibility (BIC) and Interim Individual Rationality (Interim IR).

**Proof:** We first construct the equivalent conditions for BIC and Interim IR. We construct the following Lemma.

**Lemma 1**: Under the Assumption Eqs. 4 and 5, if our auction allocation rule $q_i(v_i, v_{-i})$ could be set non-decreasing with $v_i$, we could derive the equivalence condition that the expected surplus could be written as follows:

$$\Pi_i(v_i, v_i) = \Pi_i(\underline{v}_i, \underline{v}_i) + \mathbf{E}_{v_{-i}} \int_{\underline{v}_i}^{v_i} \frac{\partial N(q_i(z, v_{-i}), z)}{\partial z} dz \tag{17}$$

**Proof:** (Necessary conditions) We first prove the Eq. 17 could be induced by the BIC and Interim IR and an extra condition, that is $\Pi(x, v_i) \geq \Pi_i(x, x)$ for $v_i \geq x$. It shows if we have a higher type, we could derive more for the same bidding. First, we could derive $\Pi_i(v_i, v_i) \geq \Pi_i(x, v_i)$ then we could derive $\Pi_i(v_i, v_i) \geq \Pi_i(x, x)$ for $v_i \geq x$.

Then we could also derive

$$\begin{aligned} \Pi_i(v_i, v_i) - \Pi_i(v_i, x) =& \mathbf{E}_{v_{-i}}[N(q_i(v_i, v_{-i}), v_i) - N(q_i(v_i, v_{-i}), x)] \\ =& \mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \int_x^{v_i} \frac{\partial r(z, y)}{\partial y} dy dz \\ \leq& \mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \int_x^{v_i} \frac{\partial r(z, x)}{\partial x} dy dz, \end{aligned} \tag{18}$$

where the inequality follows the assumption Eq. 5. Hence for all $v_i \leq x$, we have

$$0 \leq \Pi_i(v_i, v_i) - \Pi_i(x, x) \leq (v_i - x)\mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \frac{\partial r(z, x)}{\partial x} dz \tag{19}$$

Therefore $\Pi_i(v_i, v_i)$ is continuous. Then we could also derive that $\Pi_i(v_i, x)$ is a differentiable function of $x$ since $N(q(x, v_{-i}), x)$ is differentiable for $x$. Moreover, we could find out that $\Pi_i(x, x)$ is continuous and non-decreasing, hence differentiable almost everywhere. Therefore, we know from BIC that

$$v_i \in argmin_x[\Pi_i(x, x) - \Pi(v_i, x)] \tag{20}$$

Then we use the first-order condition:

$$\frac{d\Pi_i}{dx}(x, x) - \frac{\partial \Pi_i}{\partial x}(v_i, x) = 0 \ \text{ at } \ x = v_i \tag{21}$$

Then we could derive that

$$\frac{d\Pi_i}{dv_i}(v_i, v_i) = \frac{\partial \Pi_i(x, v_i)}{\partial v_i}|_{x=v_i} = \mathbf{E}_{v_{-i}} \frac{\partial N(q_i(v_i, v_{-i}), v_i)}{\partial v_i}. \tag{22}$$

Then we could derive Eq. 17 from the continuity of $\Pi_i(v_i, v_i)$ and the equation above.

(Sufficient conditions)Next, we also try to prove our Eq. 17 is sufficient conditions for BIC and IR.

First we could know a simple proposition that $\Pi_i(\underline{v}_i, \underline{v}_i) \geq 0$. It means that we could not force any bidder to participate and the expected surplus needs to be non-negative. Then if we have $\Pi_i$ satisfies Eq. 17, we could derive for $y \geq x$ that

$$\begin{aligned} \Pi_i(y, y) - \Pi_i(x, x) =& \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial N(q_i(z, v_{-i}), z)}{\partial z} dz \\ \geq& \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial N(q_i(x, v_{-i}), z)}{\partial z} dz, \end{aligned} \tag{23}$$

with $q_i(z, v_{-i})$ is set as non-decreasing in $z$.

Then we also know from the characteristics of $r$ that it is strictly increasing in $v$. We could derive that $\Pi_i(v_i, v_i) \geq 0$, which is Interim IR.

Hence, we could also know that for $y \geq x$

$$\Pi_i(x, y) - \Pi_i(x, x) = \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial N(q_i(x, v_{-i}), z)}{\partial z} dz. \tag{24}$$

We could also find out that for $y \geq x$, $\Pi_i(x, y) \geq \Pi_i(x, x)$. Thus, we could derive from the above equations that

$$\Pi_i(y, y) \geq \Pi_i(x, y) \quad y \geq x. \tag{25}$$

The almost identical argument could be made for $y \leq x$. Therefore, we could derive BIC conditions.

Actually, with our Lemma 1, we could derive an equivalent condition for Interim IR and BIC. That is to say, if our designed auction satisfies the condition Eq. 17 and the final allocation $q_i(v_i, v_{-i})$ increases with $v_i$, our auction could satisfy Interim IR and BIC then we only need to maximize the revenue of it through the allocation and payment design.

Therefore we could introduce the condition into Eq. 16 and derive the representation of the revenue for the system operator. More specifically, we could find that expected payment for bidder $i$ follows:

$$\begin{aligned} \hat{p}_i(v_i) =& \mathbf{E}_{v_{-i}}[N(q(v_i, v_{-i})) - \int_{\underline{v}_i}^{v_i} \frac{\partial N(q_i(z, v_{-i}), z)}{\partial z} dz \\ &+ (\lambda(v_i, v_{-i}) - \alpha_i)^+ q_i(x, v_{-i}) \\ &+ \phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)] - \Pi_i(\underline{v}_i, \underline{v}_i) \end{aligned} \tag{26}$$

We could also derive the expected revenue from the bidder $i$ that

$$\begin{aligned} \tilde{p}_i =& \mathbf{E}_{v_i, v_{-i}}[N(q(v_i, v_{-i})) - \int_{\underline{v}_i}^{v_i} \frac{\partial N(q_i(z, v_{-i}), z)}{\partial z} dz \\ &+ (\lambda(v_i, v_{-i}) - \alpha_i)^+ q_i(x, v_{-i}) \\ &+ \phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)] - \Pi_i(\underline{v}_i, \underline{v}_i) \\ =& \mathbf{E}_{v_i, v_{-i}}[N(q(v_i, v_{-i})) - \frac{\partial N(q_i(z, v_{-i}), z)}{\partial z} \frac{1}{\rho_i(v_i)} \\ &+ (\lambda(v_i, v_{-i}) - \alpha_i)^+ q_i(x, v_{-i}) \\ &+ \phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)] - \Pi_i(\underline{v}_i, \underline{v}_i), \end{aligned} \tag{27}$$

where $\rho_i(v_i)$ is the hazard rate defined above.

We further an indicator function $\omega_i(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i}))$ as follows:

$$\omega_i(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i})) = \begin{cases} 1 & \lambda(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i})) \leq \alpha_i \\ 0 & \lambda(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i})) > \alpha_i, \end{cases} \tag{28}$$

We also simplify $\omega_i(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i}))$ for $\omega(v_i, v_{-i})$.

Then we could also construct the final revenue that the ISO receives and we denote it as $R$. Then $R$ could be represented as follows:

$$
\begin{aligned}
R =& \mathbf{E}_{v_i,v_{-i}} \sum_{i=1}^{N} [p_i - \lambda(v_i, v_{-i})\omega(v_i, v_{-i})q_i(v_i, v_{-i}) - \phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)] \\
& - \sum_{i=1}^{N} \Pi_i(\underline{v}_i, \underline{v}_i) \\
=& \mathbf{E}_{v_i,v_{-i}} \sum_{i=1}^{N} [N(q(v_i, v_{-i}), v_i) - \frac{\partial N(q_i(z, v_{-i}), v_i)}{\partial v_i} \frac{1}{\rho(v_i)} \\
& - \alpha_i \omega(v_i, v_{-i}) q_i(x, v_{-i})] - \sum_{i=1}^{N} \Pi_i(\underline{v}_i, \underline{v}_i) \\
=& \mathbf{E}_{v_i,v_{-i}} \sum_{i=1}^{N} I^{\lambda}(q_i, v_i, \alpha_i) - \sum_{i=1}^{N} \Pi_i(\underline{v}_i, \underline{v}_i)
\end{aligned}
\tag{29}
$$

For the maximization, since we have made the proposition that $\Pi_i(\underline{v_i}, \underline{v_i}) \geq 0$, we could derive that our $\Pi_i(\underline{v_i}, \underline{v_i})$ should be set equal to 0 by the payment $\tilde{p}_i$.

Hence, to maximize our revenue, then we need to maximize the $\sum_{i=1}^{N} I^{\lambda}(q_i, v_i, \alpha_i)$ for all $v_i$ under the constraint $\sum_{i=1}^{N} q_i \leq Q$, which is denoted by our optimization problem (P2). Then we only need to find the best $\lambda$ that exactly satisfies the demand $d$ which is showed in Algorithm 1. We also show the existence for the $\lambda$ for correctness.

Finally, we need to show our optimization could be well solved with the optimality conditions Eqs. 14 and $q_i(v_i, v_{-i})$ does not decrease with $v_i$.

Firstly, we consider the cases that $\lambda$ doesn't change if the value increases, which means the price in ED does not change in this scene. It could be simply derived that function $I^{\lambda}$ is quasi-concave with Eq. 4. We could derive that for all $\alpha$ and $\lambda$:

$$
\frac{\partial I^{\lambda}}{\partial q} > 0 \rightarrow \frac{1}{\rho} < \frac{r}{\frac{\partial r}{\partial q}}
\tag{30}
$$

Then

$$
\begin{aligned}
\frac{\partial^2 I^{\lambda}}{\partial q^2} &= \frac{\partial r}{\partial q} - \frac{\frac{\partial^2 r}{\partial q \partial v}}{\rho} \\
&\leq \frac{\partial r}{\partial q} - \frac{r \frac{\partial^2 r}{\partial q \partial v}}{\frac{\partial r}{\partial q}} \\
&= \frac{r^2}{q \frac{\partial r}{\partial v}} \frac{\partial}{\partial v} \left( -\frac{q}{r} \frac{\partial r}{\partial q} \right) \\
&< 0
\end{aligned}
\tag{31}
$$

Therefore, we could know $I^{\lambda}(q, v)$ is indeed strictly quasi-concave for all $\alpha$ and $\lambda$.

Furthermore, we could derive that

$$\begin{aligned}\frac{\partial^2 I^\lambda}{\partial q \partial v} &= \frac{\partial r}{\partial v}[1 - \frac{1}{\frac{\partial r}{\partial v}} \frac{\partial}{\partial v}\left(\frac{1}{\rho}\frac{\partial r}{\partial v}\right)] \\ &= \frac{\partial r}{\partial v}\left(1 + \frac{1}{\rho^2}\frac{d\rho}{dv}\right) - \frac{1}{\rho}\frac{\partial^2 r}{\partial v^2} \\ &= \frac{\partial r}{\partial v}\frac{dJ}{dv} - \frac{1}{\rho}\frac{\partial^2 r}{\partial v^2} \\ &> 0.\end{aligned} \tag{32}$$

The last inequality follows our assumption Eq. 5 and the regularity of the distribution function. With our optimization problem (P2), we could find that the optimal solution could satisfy our Eqs. 14 through K.K.T. conditions. Since strict quasi-concavity, we could derive Eqs. 14 are also sufficient. The remaining problem is to prove the $q^*(v_i, v_{-i})$ is non-decreasing with $v_i$. If $q_i^*(v_i, v_{-i}) = 0$, it is simply that $\frac{\partial q_i^*}{\partial v}(v_i, v_{-i}) \geq 0$ since $q_i^*(v_i, v_{-i}) \geq 0$. Then if $q_i^*(v_i, v_{-i}) > 0$. we could derive from Eqs. 14 that

$$\frac{\partial I^\lambda}{\partial q}(q_i^*(v_i, v_{-i}), v_i) = \mu(v_i, v_{-i}). \tag{33}$$

Then after differentiating the last equation with respect to $v_i$, we could derive that

$$\frac{\partial^2 I^\lambda}{\partial q^2}\frac{\partial q_i^*}{\partial v_i} + \frac{\partial^2 I^\lambda}{\partial q \partial v} = \frac{\partial \mu}{\partial v_i}. \tag{34}$$

Then if $\frac{\partial \mu}{\partial v_i}$ is non-positive, we could find out that $\frac{\partial^2 I^\lambda}{\partial q \partial v}$ is positive in Eq. 32 and $\frac{\partial^2 I^\lambda}{\partial q^2}$ is negative in Eq. 31. Then $\frac{\partial q_i^*}{\partial v_i}$ is positive.

If $\frac{\partial \mu}{\partial v_i}$ is positive, it could be derived with Eqs. 14 that $\mu > 0$ since $\mu$ is Lagrangian multiplier. Then for $j \neq i$, we could derive if $\frac{\partial I^\lambda(q_j^*(v_i,v_{-i}),v_j)}{\partial q} < \mu$, $\frac{\partial q_j^*}{\partial v_i} = 0$ since $q_j^*(v_i, v_{-i}) = 0$. If $\frac{\partial I^\lambda(q_j^*(v_i,v_{-i}),v_j)}{\partial q} = \mu$, then $\frac{\partial^2 I^\lambda(q_j^*(v_i,v_{-i}),v_j)}{\partial q^2}(\frac{\partial q_j^*}{\partial v_i}) = \frac{\partial \mu}{\partial v_i}$. Then we could find that $\frac{\partial q_j^*}{\partial v_i} < 0$ due to Eq. 31. In general, we could observe that $\frac{\partial q_j^*}{\partial v_i}$ is non-positive, then with our Eqs. 14 and $\mu > 0$, we could know that $\sum_{j=1}^N \frac{\partial q_j^*}{\partial v_i} = 0$. Therefore, we could derive that $\frac{\partial q_i^*}{\partial v_i}$ is non-negative in this case. Therefore, we could prove our auction satisfies Interim IR and BIC and we could derive the maximal revenue. The specific payment $p_i$ could be found with our Eq. 27 for this scene.

Then we consider the scene that the $\lambda$ changes if $v_i$ increases. If $\mu$ does not increase, it's clear that for other generator $j$ whose $\alpha_j < \lambda$, $q_j^*$ could not decrease through our Eq. 14 and Eq. 31. Furthermore we could know that $\lambda$ could only not increase. Then it could further improve $\frac{\partial I^\lambda(q_i,v_i,\alpha_i)}{\partial q_i}$ of generator of $i$ if $\alpha_i \geq \lambda$. It is obvious that the quantity of $q_i$ could not decrease. Then if $v_i$ increases and $\mu$ increases, we discuss about it for four cases. We set the ED price as $\lambda$ and the price after $v_i$ changes to $v_i^{'}$ as $\lambda^{'}$. Then we assume $\alpha_i < \lambda$, then we could find that with $v_i$ increasing, $\lambda$ decreases. If $\alpha_i < \lambda^{'}$, $q_i$ should increase. That is because $q_j$ for $\alpha_j < \alpha_i$ should not increase because $\mu$ increases but our demand still should be satisfied, and $d - \sum_{\alpha_j < \lambda'} B_j$ increases then our $q_i^*$ needs to increase. Actually, $\alpha_i \geq \lambda^{'}$ does not hold. If it holds, we find if we decrease our $v_i^{'}$ to original $v_i$, $\mu$ should decrease and the allocation for those generators whose $\alpha_j < \lambda^{'}$ should increase since $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ for them does not change. Hence, the demand could be also satisfied in $\lambda < \lambda^{'} \leq \alpha_i$, which is contradict to our assumption.

Then we discuss about the scene that $\alpha_i \geq \lambda$. Then we first focus on the scenario $\lambda^{'} \leq \alpha_i$. Then we could find $\frac{\partial I^\lambda(q_i,v_i,\alpha_i)}{\partial q_i}$ function could increase with $v_i$ and for other generator $j$, their $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ will not increase. Then according to Eq. 14 and 31, the allocation for $i$ $q_i^*$ should increase when $\mu$ increases. Then we also need to show the other case $\lambda^{'} > \alpha_i$ could not hold. If it holds, we could pay attention to the generator with $\alpha_j \geq \lambda'$. Their $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ could not change. If we also make our

$v_i^{'}$ decreases to $v_i$, $\mu$ would decrease. Then the quantity of certificate for them would increase and the left certificate could decrease. We know our demand could satisfies at $\lambda^{'}$, if the left certificate decreases, $\lambda$ should increase and we could derive $\alpha_i < \lambda^{'} \leq \lambda$, which is contradict to our assumption. Therefore, in general, we could derive $q_i^*$ could be monotone even if $\lambda$ changes.

Actually, there could be lots of interesting findings through our framework. We could find out that in our auction, $I^{\lambda}$, which could reflects the virtual demand for the generators could be lower if the generator participate in the generation. Actually, it would make sense that the generators that satisfy the demand with a small cost would have a higher value. But it is still interesting that the generators that do not take part in the generation would have a higher incentive. Actually, it could be found in our proof that if the generator takes part in the generation, it could be forced by the auctioneer with collecting extra payment from the extra profit from the generation. Since the process could be forced, the incentive of the generator could reduce. Actually, from the perspective of the system operator, through the process, we could attach more importance to the generator that only wants to make a contribution. In other words, we would appreciate the buying of the high cost generator because it could have great emission reduction process and its buying only aims to devote for the Carbon Capturing. Otherwise, the low cost generator could pay more attention to have more generation through the auction, we need to relatively pay not much attention to it.

# 4 Competition Analysis for Limited Generation

In this part, we take the load generation constraint into consideration. We assume for each generation $i$, the maximal load could be denoted as $G_i$.

We need to consider progress about the above mechanism from two perspectives. First, we could modify our optimization problem (P2) with the load constraints as follows:

$$\begin{aligned} (P3) \max_{q_i} &\sum_{i=1}^{N} I^{\lambda}(q_i, v_i, \alpha_i) \\ s.t. \quad &\sum_{i=1}^{N} q_i \leq Q \\ 0 \leq q_i &\leq G_i - B_i \end{aligned} \tag{35}$$

Assuming the generators need to compete for the green certificate, in other words, it is rare for all the generators, we could conduct this allocation according to the optimization problem (P3).

Then to guarantee the truthfulness, we further need to show our allocation with (P3) is monotone.

**Theorem 3:** If we assign the certificate according to the optimization problem (P3) to replace (P2) in Algorithm 1 when certificate is rare as we mentioned above, we could guarantee $q_i^*$ could be non-decreasing if $v_i$ increases.

**Proof:** We first need to show as our Theorem 1 that there could be a unique $\lambda$ that satisfies the demand. We could order the generator as their margin cost as we do in Theorem 1's proof.

Without loss of generality, we assume $\lambda = \alpha_K$, we could not satisfy the demand. Then we try $\lambda = \alpha_{K+1}$ and we regard original allocation for generator $i$ as $q_i$ and new one as $q_i^*$. Then for generator $K$, its $I^{\lambda}(q_K, v_K, \alpha_K)$ could decrease by $\alpha_i q_K$. We could find out with $\lambda$ increases, $\mu$ for K.K.T conditions should not increase. That is because according to the K.K.T conditions for problem (P3), we could derive if $\mu$ increases, the quantity allocated for other generator should not increase according to the Eq. 31. Then for generator $K$ the quantity should not increase also. Then we could derive that our allocation should be lower than before. And it means our $\mu$ should not increase since $\mu \geq 0$. Then we know $\mu$ should decrease and we could know that the allocation for generator $j$ whose $j < K$, their allocation could not decrease. There could be two cases. First, if $q_j = G_j - B_j$, the decrease could make the lagrangian multiplier of load constraint for $j$ $\tau_j$ increase and $q_j$ does not change. Otherwise, $q_j$ would increase due to $\tau_j = 0$ and $\mu$ decreases.

Therefore, we could know $\sum_{j<K} q_j < \sum_{j<K} q_j^*$. Then we could derive the maximum demand that could be satisfied $\sum_{i=1}^{K-1} B_j + q_j < \sum_{i=1}^{K} B_j + q_j$ while $\lambda$ increases from $\alpha_K$ to $\alpha_{K+1}$. We could find our maximum demand that could be satisfied is monotone increasing and we know we could satisfy the demand if all the generators take part in the generation and all the certificate is allocated. Therefore, we could derive our $\lambda$ should exist and be unique.

Then we come back to the proof of our Theorem 3. We need to discuss two cases. If $\lambda$ does not change with $v_i$ increases, then all $I^{\lambda}$ could not change except for $i$. Then we assume the $v_i$ increases $\eta$. Then from our Eqs. 31 and 32, we could derive that $I^{\lambda}$ is also quasi-concave. Then we could derive a new optimal conditions as follows:

$$\frac{\partial I^{\lambda}}{\partial q}(q_i^*(v_i, v_{-i}), v_i) = \mu(v_i, v_{-i}) + \tau_i, \tag{36}$$

where $\tau_i$ is the Lagrangian multiplier of the constraints $q_i \leq G_i$. If $q_i^*(v_i, v_{-i}) \leq G_i - B_i$, we could derive $\tau_i = 0$. Then we could analyse as we do in Theorem 2. If $\mu$ is increase, it could be obvious from the aforementioned proof. If $\mu$ is non-increasing and there do not exist generators whose $\tau$ is not 0, we could derive the exact same results as Theorem 2's proof. If there exist some generators whose $\tau = 0$, we could derive that the quantities of them could not increase and it follows the same results that $q_i$ could increase.

Then if $q_i^*(v_i, v_{-i}) = G_i - B_i$, then we assume there exist a $v_i^{'} > v_i$ that makes $q_i^*$ decreases to $q_i^{*'}$. We also define other generation's allocation as $q_j^{*'}$ and $j \neq i$. We could derive

$$\begin{aligned} I^{\lambda}(q_i^{*'}, v_i^{'}, \alpha_i) + \sum_{j \neq i} I^{\lambda}(q_j^{*'}, v_j, \alpha_j) &> I^{\lambda}(q_i^{*}, v_i^{'}, \alpha_i) + \sum_{j \neq i} I^{\lambda}(q_j^{*}, v_j, \alpha_j) \\ &> I^{\lambda}(q_i^{*}, v_i^{'}, \alpha_i) - I^{\lambda}(q_i^{*}, v_i, \alpha_i) + I^{\lambda}(q_i^{*'}, v_i, \alpha_i) + \sum_{j \neq i} I^{\lambda}(q_j^{*'}, v_j, \alpha_j), \end{aligned} \tag{37}$$

Then we could derive that

$$I^{\lambda}(q_i^{*}, v_i, \alpha_i) + I^{\lambda}(q_i^{*'}, v_i^{'}, \alpha_i) - I^{\lambda}(q_i^{*'}, v_i, \alpha_i) - I^{\lambda}(q_i^{*}, v_i^{'}, \alpha_i) > 0, \tag{38}$$

Then

$$\frac{I^{\lambda}(q_i^{*}, v_i, \alpha_i) + I^{\lambda}(q_i^{*'}, v_i^{'}, \alpha_i) - I^{\lambda}(q_i^{*'}, v_i, \alpha_i) - I^{\lambda}(q_i^{*}, v_i^{'}, \alpha_i)}{(q_i^{*} - q_i^{*'})(v_i - v_i^{'})} < 0, \tag{39}$$

If $\lambda$ does not change, the $I^{\lambda}$ is continuous and with the Lagrange mean value theorem, we could derive there could exist $(q, v)$ that makes $\frac{\partial^2 I^{\lambda}}{\partial q \partial v} < 0$ As we show in Eq. 32, we find the contradiction. Therefore, the monotonicity could hold when $\lambda$ does not change.

Then if $\lambda$ changes, we also need to prove the monotonicity. We need to analyze the cases for $q_i^*(v_i, v_{-i}) = G_i - B_i$ and $q_i^*(v_i, v_{-i}) < G_i - B_i$ as we show above. We first discuss for $q_i^*(v_i, v_{-i}) < G_i - B_i$. In the following part, we denote the new ED price as $\lambda^{'}$ and new value as $v_i^{'}$ and new quantity as $q_i^{*'}$ for generator $i$.

We first suppose $\mu$ is not increasing. Then we could find that $q_i^{*'}$ should not decrease. Since $\mu$ is not increasing, for generator $j$ whose $\alpha_j < \lambda^{'}$, their $q_j^{*'}$ should be non-decreasing according to the K.K.T conditions of (P3). Then we could derive that $\lambda^{'} < \lambda$ and for generator $i$, its $\frac{\partial I^{\lambda}(q_i, v_i, \alpha_i)}{\partial q_i}$ could be increasing according to Eq. 32. Then according to Eq. 31 we derive that $q_i^{*'}$ could be non-decreasing. Then we discuss about the cases that $\mu$ is increasing. We also take four cases. We suppose $\alpha_i < \lambda$. Then we also assume $\alpha_i < \lambda^{'}$. Then we could derive that for generator $j$ whose $\alpha_j < \alpha_i$, their $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ would not change but $\mu$ increases. Therefore, their $q_j^{*'}$ should not increase but our demand could also be satisfied and $d - \sum_{\alpha_j < \lambda^{'}} B_j$ increases, then $q_i^{*'}$ should increase. Then we also point out the impossibility that $\lambda^{'} \leq \alpha_i$. If it holds, for generator $j$ whose $\alpha_j < \lambda^{'}$, their $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ would not change but $\mu$ increases and the certificate quantity would decrease and the demand could not be satisfied. Then we suppose $\alpha_i \geq \lambda$. Then assume $\lambda^{'} \leq \alpha_i$. Then we could derive that for other generator $j$ their $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ would not increase. According to Eq. 31, we could derive when $\mu$ increases, their allocation should decrease and $q_i^*$ should increase. Then we also show $\alpha_i < \lambda^{'}$

is impossible. If it holds, we could derive for generator $j$ whose $\alpha_j \geq \lambda^{'}$, their allocation should decrease and the total allocation for generators $j$ whose $\alpha_j < \lambda^{'}$ should increase, which is higher than the original allocation for generators $j$ whose $\alpha_j < \lambda$. Then we could derive that demand could be over satisfied, showing the $\lambda^{'}$ could not be such high. Therefore, we could derive that our monotoncity could hold when $q_i^*(v_i, v_{-i}) < G_i - B_i$.

Then we discuss about the scene when $q_i^*(v_i, v_{-i}) = G_i - B_i$. We need to show if $v_i$ increases, $\lambda$ could not change in this case. As we show above, if $\lambda$ does not change, we could derive that our $q_i^*$ could not change if $v_i$ increases and other generator's allocation will not change as well. Then we could derive that we could have a possible solution with ED price $\lambda$.

As we showed above, $\lambda$ should be unique. Therefore, we could claim that our allocation would not change in this case. In general, we have showed the monotoncity for problem (P3).

After discussion about the competitive scenes, we also need to modify our mechanism to be adaptive for the non-competitive cases, where $\sum_{i=1}^{N} G_i - B_i < Q$. Then the problem should be the payment design. Actually, we could find it a specific case of the optimization problem (P3). Therefore, we could follow the allocation rule for competitive scenes and it could also have the same characteristics.

Finally, we again state our modified mechanism as the following Algorithm 2:

**Algorithm 2:** Green Certificate Auction Considering ED Process with load constraint

**Input:** The generator $i$'s cost $\alpha_i \; \forall i \in [N]$;
The generator $i$'s bidding type $v_i \; \forall i \in [N]$;
The generator $i$'s Maximum Load $G_i \; \forall i \in [N]$ and Maximum load without certificate $B_i$;
**Output:** The final allocation $q_i \; \forall i \in [N]$;
The price $\lambda$ for ED process;
The payment $p_i \; \forall i \in [N]$;

```
Initialize R = 0
if Σ_{i=1}^{N} G_i − B_i < Q then
    q_i = G_i − B_i
    Decide λ through ED process and decide p_i according to Eq. 15
    return (q_i, p_i) ∀i; λ;
else
    for λ ∈ {α_1, ..., α_N} do
        Solve the optimization problem (P3) and Eq. 15 with G_i and B_i to derive (q_i^λ, p_i^λ) ∀i.
        Use final auction allocation results and price λ to conduct ED process
        if d is exactly satisfied then
            Calculate the final revenue R_t for auctioneer;
            if R_t ≥ R then
                q_i = q_i^λ, p_i = p_i^λ ∀i; R = R_t
            end if
        end if
    end for
    return (q_i, p_i) ∀i; λ;
end if
```

## 5 Sample Complexity for optimal Green Certificate Auction

We next focus on the scenarios that we could not clearly identify the willingness of dedication. Furthermore, we actually could not derive a concise distribution function for each type $v_i$. Therefore, we need to learn its type distribution from the history samples and the problem rises how many samples we need for us to achieve an approximate optimal. Therefore, in this part, we mainly discuss about this problem.

We need to first reformulate our auction. We first construct our auction as a function $h$ whose inputs are different constant cost $\alpha_i$ and variable $v_i \; \forall i \in [N]$. Then we could know the maximal revenue should be lower than $\max_{q_i} \sum_{i=1}^{N} N(q_i, \overline{v}_i)$, which could be denote as a constant $C_1$. We also assume our uniform price function $r(q,v)$ satisfies that $|\frac{\partial r}{\partial v}|$ and $|\frac{\partial^2 r}{\partial v^2}|$ are bounded in the interval $[\underline{v}_i, \overline{v}_i]$. We also need to add assumptions about the distribution function $F_i$ and $f_i$. We assume $f_i > 0$ in the interval $[\underline{v}_i, \overline{v}_i]$ and $f_i$ is differentiable. Then we could derive that $\frac{1}{\rho_i(v_i)}$ could be Lipschitz continuity with parameter $L_3$.

Then we could use it to make our function $h$ be a map from $\mathbf{v} = \{v_i, \forall i \in [N]\}$ to $[0,1]$.

Then as we discuss above, we know $\mathbf{v}$ satisfies a distribution $F = F_1 \times F_2 \times ... \times F_N$ which $F$ denotes a product distribution, which means all the types for each bidder is independent. Then we could derive $h(F)$ as follows:

$$h(F) = \mathbf{E}_{\mathbf{v}\sim F}[h(\mathbf{v})] \tag{40}$$

Then we define the mechanism could be chosen from a hypothesis class, which could be denoted by $\mathscr{H}$. Then we could denote $OPT_{\mathscr{H}}(F)$ as the optimal expected revenue. Specifically,

$$OPT_{\mathscr{H}}(F) = \sup_{h\in\mathscr{H}} h(F). \tag{41}$$

Actually, our samples are taken from the distribution $F$. Then we denote $E_i$ as the uniform distribution over $i-th$ coordinate of the samples, and we could also define $E = E_1 \times E_2 \times ... \times E_N$. The sample complexity for our hypothesis class $\mathscr{H}$ could be the minimum number of samples for any distribution $F$, we could find a mechanism $h$ so that

$$h(E) \leq OPT_{\mathscr{H}}(F) - \epsilon, \tag{42}$$

with probability $1-\delta$, where $\epsilon$ is a small number in $[0,1]$.

Next, we could give our theorem for the upper bound of the sample complexity for our auction as follows:

**Theorem 4**:In our proposed Green Certificate Auction with $N$ generators, the sample complexity is at most $O(\frac{N^2}{\epsilon^3} log\frac{1}{\delta})$ when $\epsilon$ is enough small.

**Proof:** To prove the theorem, we need to first construct an auxiliary distribution to conduct discretization.

We first construct a finite support for each $[\underline{v}_i, \overline{v}_i]$ by the interval size of $\zeta$ as $\{\underline{v}_i, \underline{v}_i + \zeta, ..., \overline{v}_i\}$ with size of $(\overline{v}_i - \underline{v}_i)\zeta$.

We construct a discrete distribution by rounding the values from the distribution $F_i$ to the closest multiple of $\zeta$ that is higher than the original values for $\alpha_i < \lambda$, and lower than original values otherwise. Then we denote it the new distribution as $F_i^{'}$.

Then we have the following Lemma 2.

**Lemma 2** For this new distribution $F^{'}$, we have

$$OPT(F^{'}) \geq OPT(F) - O(\zeta), \tag{43}$$

where $OPT(F)$ denotes the optimal auction revenue under distribution $F$.

**Proof:**

We first let $M$ as the optimal mechanism with respect to $F$, which is our auction designed in Algorithm 1. Next, we construct a quantiles $\xi_i$ for each $v_i$. $q_i$ satisfies that $v_i(\xi_i) = \inf\{v : F_i(v) \geq \xi_i\}$ for a certain distribution $F_i$. Then if we know $v$, we let $\underline{l}_i(v_i) = \sup_{v<v_i} F_i(v)$ and $\overline{l}_i(v_i) = F_i(v_i)$. If $\underline{l}_i(v_i) = \overline{l}_i(v_i)$, $\xi_i$ could be mapped as it. Otherwise, $\xi_i$ could be uniformly sampled from $[\underline{l}_i(v_i), \overline{l}_i(v_i)]$. After deriving the mapping from $v$ to $\xi$, we construct a mechanism $M^{'}$ for the distribution $F^{'}$ as follows:

- 1. Given a rounded value $\mathbf{v}^{'}$, we map it to get its quantile $\xi$ for each coordinate based on the distribution $F^{'}$.
- 2. Let $\mathbf{v}^{''}$ be the values that correspond to $\xi$ with respect to the distribution $F$.
- 3. Use the mechanism $M$ with the values $\mathbf{v}^{''}$ to conduct the allocation.

Then it is clear that the allocation is monotone for $\mathbf{v}^{'}$, and our allocation rule in Lemma 1 could guarantee that there could exist a payment rule that makes $M^{'}$ truthful.

Then we could couple all the randomness by sampling the quantiles $\xi$. Given any $\xi$, we could know our $M^{'}$ and $M$ returns the same allocation. For the payment, we could know $\rho_i(v_i(\xi_i))$ and $\rho_i(v_i^{'}(\xi_i))$ could be the same. Then we know the rounding process could lead to a difference for value by at most $\zeta$ and we only need to know how this influences the final revenue.

We need to focus on the characteristics of our function $N(q, v)$ about $v$. We could derive that $\frac{\partial r(q,v)}{\partial v} > 0$ and $\frac{\partial^2 r(q,v)}{\partial v^2} \leq 0$ from our assumptions. It shows that our $N(q, v)$ satisfies Lipschitz continuity and we assume the parameter is $L_1$. For $\frac{\partial N(q,v)}{\partial v}$, we could also derive it satisfies Lipschitz continuity and we assume the parameter is $L_2$.

We denote the optimization problem (P4) with values $\mathbf{v}^{'}$ as follows:

$$\begin{aligned} (P4) \max_{q_i} \sum_{i=1}^{N} I^{\lambda}(q_i, v_i^{'}, \alpha_i) \\ s.t. \quad \sum_{i=1}^{N} q_i \leq Q \end{aligned} \tag{44}$$

Based on the Lipschitz continuity, we could find the following inequalities hold:

$$N(q, v) \geq N(q, v^{'}) - L_1 \zeta Q \tag{45}$$

$$\frac{1}{rho(v)} \leq \frac{1}{rho(v^{'})} + L_3 \zeta \tag{46}$$

$$\frac{\partial N(q, v)}{\partial v} \leq \frac{\partial N(q, v^{'})}{\partial v} + L_2 \zeta Q \tag{47}$$

Then we have $\frac{\partial N(q,v)}{\partial v} > 0$ and its bound $B_2$. We also have $\frac{1}{\rho_i(v_i)} > 0$ and its uniform bound $B_1$.

$$\begin{aligned} \sum_{i=1}^{N} I^{\lambda}(q, v^{'}, \alpha_i) &\geq \sum_{i=1}^{N} I^{\lambda}(q, v, \alpha_i) - N L_1 \zeta Q - N B_1 L_2 \zeta Q - N L_3 B_2 \zeta \\ &\quad + N B_1 B_2 L_2 L_3 \zeta^2 Q \\ &\geq \sum_{i=1}^{N} I^{\lambda}(q, v, \alpha_i) - N L_1 \zeta Q - N B_1 L_2 \zeta Q - N L_3 B_2 \zeta, \end{aligned} \tag{48}$$

where we could find out that we back to the optimization problem (P2).

**Remark:** Actually, there could be some extreme scenarios, which could make $\lambda$ change and make the optimal revenue change. Then if we round the value, we could know values for the generators that take part in the generation could rise while others' could go down. Then we could derive that the allocation for generators $j$ whose $\alpha_j < \lambda$ could not decrease. If the allocation decreases, reflecting that allocation for other generators $i$ whose $\alpha_i \geq \lambda$ should increase. Since other $\frac{\partial I^{\lambda}(q_i, v_i, \alpha_i)}{\partial q_i}$ should decrease then $\mu$ should decrease. Then for generator $j$, their allocation need to be raised, where contradiction occurs. Then we could find if we take rounding, we could find our mechanism could satisfy more demand than original values, which shows the ED price should be higher. Without loss of generality, we could set the ED price for original value as $\lambda$ and $\lambda^{'}$ as the price after rounding. Then we could derive that $d > \sum_{\alpha_j < \lambda} q_j + B_j$. We make that $d = c + \sum_{\alpha_j < \lambda} q_j + B_j$. Then we could find if we have enough small $\zeta$ we could derive $\lambda^{'} = \lambda$. As we show in Theorem 2, $\frac{\partial q_j}{\partial v_j}$ is non-negative. Then since $q_j$ is bounded, it could exist an uniform bound for $\frac{\partial q_j}{\partial v_j}$. Then we could find out that if our $\zeta$ becomes enough smaller, the change of allocation for generators whose $\alpha_j < \lambda$

should be enough small, which could make $\lambda$ not change and further make our Eq. 48 hold. Here, we assume our $\epsilon$ is enough small and we could divide for a enough small $\zeta$.

We denote the optimal solution for (P4) as OPT(P4), we could derive that

$$OPT(F) - NL_1\zeta Q - NB_1L_2\zeta Q - NL_3B_2\zeta \leq OPT(P4) \leq OPT(F^{'}). \tag{49}$$

After we show the Lemma 1, we could next conduct Theorem 1 in [24]. We first introduce this theorem as our Lemma 3.

**Lemma 3 (Theorem 1 in [24]):** For any distribution $F^{'}$ on a finite set $\mathbf{v}$ such that $|v_i| \leq \kappa$ for all $1 \leq i \leq N$, suppose for some sufficiently large constant $C_2 > 0$, the number of samples is at least $C_2 \cdot \frac{Nk}{\epsilon^2} log\frac{1}{\delta}$, then with probability $1 - \delta$, for any $\mathbf{v} \to [0, 1]$, we have

$$|h(F) - h(E)| \leq \epsilon, \tag{50}$$

where $E$ is the empirical distribution we define before.

Then we could map the auction results to $[0, 1]$ with constant $C_1$. Then we could derive that

$$OPT_{\mathscr{H}}(F) - \frac{NL_1Q + NB_1L_2Q + NL_3B_2}{C_1}\zeta \leq OPT_{\mathscr{H}}(F^{'}) \tag{51}$$

With our Lemma 3, if we denote the sample number as $m$, we could derive that

$$|h(F^{'}) - h(E)| \leq \sqrt{C_2 \cdot \frac{N\kappa}{m} log\frac{1}{\delta}}, \tag{52}$$

with probability $1 - \delta$.

We also know $\kappa = \max_i \frac{\overline{v}_i - \underline{v}_i}{\zeta} = \frac{C_3}{\zeta}$.

Then we could derive that

$$\begin{aligned} OPT_{\mathscr{H}}(F) - \frac{NL_1Q + NB_1L_2Q + NL_3B_2}{C_1}\zeta &\leq OPT_{\mathscr{H}}(F^{'}) \\ &\leq OPT_{\mathscr{H}}(E) + \sqrt{C_2 \cdot \frac{NC_3}{m\zeta} log\frac{1}{\delta}} \end{aligned} \tag{53}$$

Then we could derive that

$$OPT_{\mathscr{H}}(E) \leq OPT_{\mathscr{H}}(F) - \frac{1}{3}\left(\frac{(NL_1Q + NB_1L_2Q + NL_3B_2)C_2C_3N}{4C_1m} log\frac{1}{\delta}\right)^{\frac{1}{3}}, \tag{54}$$

where we take

$$\zeta = \left(\frac{NC_2C_3C_1^2}{4m(NL_1Q + NB_1L_2Q + NL_3B_2)^2} log\frac{1}{\delta}\right)^{\frac{1}{3}}, \tag{55}$$

where we could find if our amount of samples become larger, we could take higher resolution to make the approximate error smaller.

Then we could find the sample complexity of our problem could be $O(\frac{N^2}{\epsilon^3} log\frac{1}{\delta})$.

# 6 Numerical Studies

In this section, we try to conduct numerical studies with our proposed framework.

## 6.1 Settings for numerical studies

We first the true generator margin cost data and capacity data in EIA-860 in U.S. Energy Information Administration's data [25]. Then we randomly assign the demand $d$ and the quantity of the total certificates $Q$ and guarantee the $d \leq \sum_{i=1}^{N} B_i + Q$. Then we also set the value scale for each generator randomly. Then we try to take the values from some scale with a certain distribution (uniform or truncated normal). For our function $r(q,v)$, we set it as the form as the polynomial form of the $q$ and $v$ as follows:

$$r(q,v) = g - a_1 q^2 + b_1 vq - a_2 v^2 - a_3 q + b_2 v, \tag{56}$$

where we could check it satisfies the conditions we mentioned in Section 2. Then we could also derive the Lipschitz constant $L_1, L_2, L_3$ and the corresponding bound $B_1, B_2$. Then we could also derive $C_1$ and $C_3$ from the value and $Q$. Then from [24], we could derive the constant $C_2$.

In the following part, we try to conduct two types of numerical studies for our proposed auction. Since our proposed auction pay attention to the one-shot trading and we have also showed its effectiveness, we try to extend it to the multi-shot trading. We propose some strategies to optimize the revenue heuristically and verify its performances empirically. Then next we could conduct our numerical studies to verify the effectiveness of our proposed sample complexity bound. The values scale is uniform choose and for the truncated normal distribution, the variance could be set as $\frac{1}{8}$ of the scale length and we also set it as symmetry. The probability $\delta$ in our sample complexity bound could be set as $0.1$ for error analysis. For the probability, we also set the error $\epsilon$ as different values to find the relationship between the error probability and the size of samples. For each size of samples, we repeat the experiments for $800$ times to derive a convincing results.

### 6.2 Extension for multi-shot scenarios

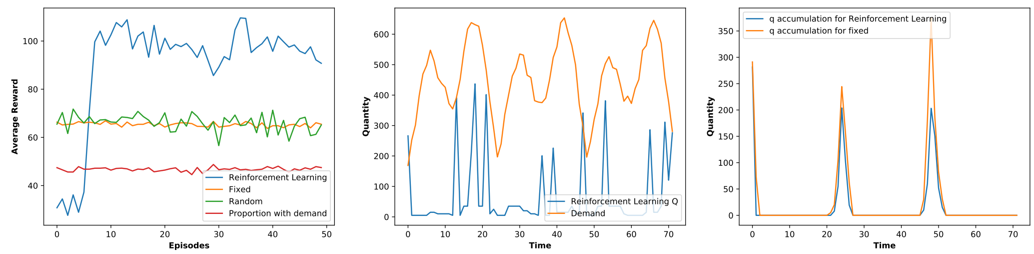
In this part, we try to empirically improve our framework to a more realistic scenarios with multi-period auction for each dispatch decision time. The main difference between one-period and multi-period comes from that the certificate' would accumulate if the agents do not use them in the current period and influence the auction for next period. Here, we assume the participate only need to bid its value at the beginning period. Then if he has no knowledge for the coming demand, our auction would also achieve truthfulness. Then what the system operator could determine is the pieces of the certificates they want to auction. We know if the system operator releases too much for each time, the participators that have already some certificates might not be willing to purchase since the margin cost' reduction. Different from the one-period, we also take the cost for releasing the certificate into consideration because in one period we could not care about whether the certificate could be used up but in multi-period it has accumulative effects and we could not release unlimited certificates. In the real world, this releasing cost represents the punishment if the excessive certificate has been released but we can not collect it for carbon reduction.

This problem could be challenging since we could not derive the true demand for next period. Therefore, we take some heuristic strategies to propose some possible solutions and also emphasize the significance to determine the release. The network is a simple 3-layer MLP since our input dimension is not so large.

The strategies we consider is fixed release, random release, release proportional last demand and reinforcement learning decision. In the reinforcement learning, we conduct a simple deep Q-learning framework with the input state of last certificate remaining for all players and the last 6 periods' demand. Our action comes from 100 discrete levels for certificate releasing, which is search in 10,100,1000 levels. We also set the learning rate at 0.001 and the batch size at 64 in memory capacity 400. We take 3 days with one hour resolution as an episodes to train and verify the effectiveness of our strategies and results are showed as following figures.

From Fig. 1(a), we could view that our proposed reinforcement learning outperforms with the training episode increasing. A strategic release could earn about twice for the fixed or random release. Then in Fig. 1(b), we could find the release of the certificate increases often when the demand will increase next. It shows this strategy tends to store more spare certificate when the tendency of demand is slightly increasing. We could also view if we response after the demand it could cause much cost in Fig. 1(a). Then finally, we observe a certain players' certificate remaining. We could find that most of time the players take part in the dispatch and the certificate could be used for generation. But there could be some time like the late at night the demand decreases sharply when it would posses much remaining certificates. It again emphasize the importance that we need to reduce these remaining useless certificates that could influence the maximal revenue of next period's auction. We could also view in Fig. 1(c), our reinforcement learning strategy successfully decreases this remaining value.

From the results, we could find the effectiveness of our proposed reinforcement learning strategies compared with other simple strategies. This framework could be a simple solution in the real



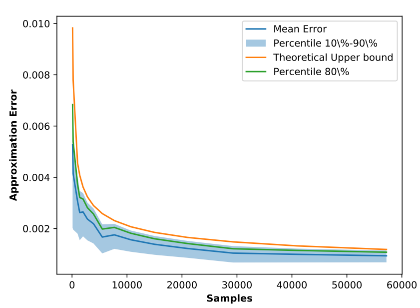

(a) The reward for different strategies (b) The strategic certificate release for reinforcement learning (c) The certificate remaining for a single participant

Figure 1: The multi-period auction results

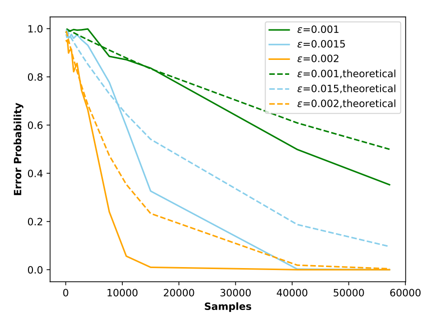
implementation of our auction in multi-period scenes and determine a suitable release for the quantity of the certificate. On the other hand, we could find that if we choose the certificate quantity arbitrarily, we might cause more remaining certificate and lead to the revenue loss or the punishment caused by the excessive release of carbon emission certificate. Therefore, if we want to make use of auction into multi-period, we should make a plan for the release for each period in detail.

### 6.3 Results for our numerical studies

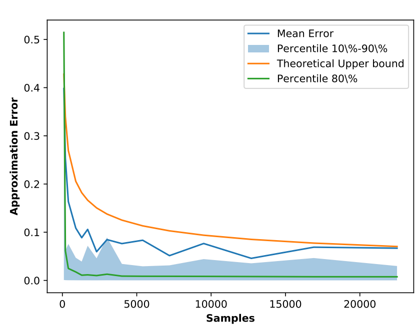
We show our numerical results in our Fig. 2. From the results, we first could find that, with the number of samples increases, we will get the results that are closer to the original one for both distribution. The error would decrease sharply with the size of the samples increasing. In Fig. 2(a),



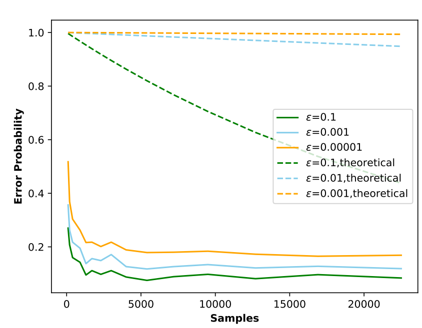

(a) Mean Error for Uniform Distribution




(b) Error Probability for Uniform Distribution




(c) Mean Error for Normal Distribution




(d) Error Probability for Normal Distribution

Figure 2: The error analysis with different size of samples

we could find that our theoretical curve is very close to the percentile of 90%, which reflects tightness of our proposed error bound in this case. Then, for the normal distribution in Fig. 2(c), we could find our theoretical is also higher than the 90% percentile and empirically, it could be closer to the mean error, which shows that our theoretical results could also represents the mean error sometimes. Then we could pay attention to the error probability. It is obvious that when the samples' numbers increase, the error probability could derive and we could have more accurate results. Moreover, we could find that when we improve the error requirement, the error probability could increase. We take different thresholds and view the probability that the errors for the approximate auction with empirical distribution. In Fig. 2(b), we could find out that our theoretical upper bound could better describe the changes of the error probability. In Fig. 2(d), we could find our upper bound is loose and the actual error could be smaller, which means that we may not need much samples and we could approach good approximation performance empirically. Then we could also compare two distribution. We could find that the results for normal distribution could be worse than the uniform distribution. That is because we need to afford more if we misjudge the hazard rate for normal distribution and our auction would also calculate a less accurate virtual demand.

As we showed in Fig. 2, we could find our theoretical bound could also be not tight. That is because we actually conduct larger scaling in Eq. 48 and we also use the uniform bound and Lipschitz constant to describe the uniform characteristics of the function, which could be rough and further cause the untightness.

From our results, we could view the effectiveness of our proposed theoretical sample complexity analysis and corresponding approximate auctions. We could measure the willingness of the generators without knowing any prior knowledge and we could also derive a relatively good performance with the empirical distribution from samples. With the scale of the samples increasing, the results could be better. In practice, we could make use of our proposed theoretical to know how much error the auctioneer could have possibly in each round with different numbers of samples. Then the auctioneer can also decide the number of questionnaire to ask for the values from the homogeneous participants after determining the possible approximate error that it can tolerate. Then once we know more about the willingness's distribution, it could be possible to design another reward mechanism for the contribution that each participants could make, which could be reflected in his value.

# 7 Conclusion

With the call for the carbon emission reduction and the carbon capture technology development, we propose a framework for the green certificate auction. Our proposed auction considers the following economic dispatch and derive the truthfulness and optimality. To better describe the willingness of the generators contributing to the carbon neutrality, we next propose a sample complexity to assist our auction and derive the upper bound of the number of samples we need to derive a near optimal approximate result. This work could be extended in many ways. The a tighter upper bound and the lower bound of sample complexity could be further considered. The continuous complex marginal cost function could need studied in detail as well as other value functions that do not satisfy our proposed assumptions. Another future work is the reward mechanism according to actual contribution that the participants make in expectation, which could be related to their values of willingness.

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# Checklist

The checklist follows the references. Please read the checklist guidelines carefully for information on how to answer these questions. For each question, change the default **[TODO]** to [Yes] , [No] , or [N/A] . You are strongly encouraged to include a **justification to your answer**, either by referencing the appropriate section of your paper or providing a brief inline description. For example:

- Did you include the license to the code and datasets? [Yes] See Section **??**.
- Did you include the license to the code and datasets? [No] The code and the data are proprietary.
- Did you include the license to the code and datasets? [N/A]

Please do not modify the questions and only use the provided macros for your answers. Note that the Checklist section does not count towards the page limit. In your paper, please delete this instructions block and only keep the Checklist section heading above along with the questions/answers below.

1. For all authors...
   (a) Do the main claims made in the abstract and introduction accurately reflect the paper's contributions and scope? [Yes] See our abstract and Section 1.
   (b) Did you describe the limitations of your work? [Yes] See Section 1.
   (c) Did you discuss any potential negative societal impacts of your work? [No]
   (d) Have you read the ethics review guidelines and ensured that your paper conforms to them? [Yes]
2. If you are including theoretical results...
   (a) Did you state the full set of assumptions of all theoretical results? [Yes] See Section 2.
   (b) Did you include complete proofs of all theoretical results? [Yes] See Section 3.
3. If you ran experiments...
   (a) Did you include the code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL)? [No] The code and the data are proprietary.
   (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they were chosen)? [Yes] See Section 6.
   (c) Did you report error bars (e.g., with respect to the random seed after running experiments multiple times)? [Yes] See Section 6 for Fig. 2(a) or 2(c).
   (d) Did you include the total amount of compute and the type of resources used (e.g., type of GPUs, internal cluster, or cloud provider)? [No] Simple personal laptop can run the experiments.
4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
   (a) If your work uses existing assets, did you cite the creators? [Yes] See Section 6.
   (b) Did you mention the license of the assets? [Yes] See Section 6.
   (c) Did you include any new assets either in the supplemental material or as a URL? [No] We do not use new assets.
   (d) Did you discuss whether and how consent was obtained from people whose data you're using/curating? [No] We just use the open dataset.
   (e) Did you discuss whether the data you are using/curating contains personally identifiable information or offensive content? [No] We just use the open dataset.
5. If you used crowdsourcing or conducted research with human subjects...
   (a) Did you include the full text of instructions given to participants and screenshots, if applicable? [Yes]
   (b) Did you describe any potential participant risks, with links to Institutional Review Board (IRB) approvals, if applicable? [Yes]
   (c) Did you include the estimated hourly wage paid to participants and the total amount spent on participant compensation? [Yes]

# A Appendix

Optionally include extra information (complete proofs, additional experiments and plots) in the appendix. This section will often be part of the supplemental material.